Dynamic Optimal Adjustment of Barrier to Learning Externality*

Yong Wang†

January, 2006

Abstract

This paper builds a theoretical model where a benevolent central government of the developing economy tries to maximize the total welfare by optimally adjusting down the barrier to learning externality in human capital,which mainly captures the dynamic change of the institution typically taken as given in standard growth literature. To this end, we characterize the optimal adjustment scheme specifying the optimal timing, size, and number of the adjustment and see how this would affect the growth experience of this economy. Our analysis mainly focuses on the deterministic learning with or without conditional foreign aid. A full characterization is obtained when only one adjustment is allowed, and partial characterization is provided for multiple adjustment with different budget constraints.

Key Words: Institutional Barrier Ajustment , Human Capital Externality, Growth

*I am very grateful to Lars Peter Hansen, Fernando Alvarez, Monika Piazessi , Thomas Chaney, William Fuchs, Justin Lin, and Balazs Szentes for very kind and helpful suggestions and discussions at different stages of this project. I also thank many other seminar participants of the capital theory working group and the macroeconomic dynamics working group at the University of Chicago, CCER at Peking University, Fudan University, Shanghai University of Finance and Economics for their comments. My special thanks go to Robert E.Lucas,Jr. and Nancy L. Stokey, both of whom offered very kind and detailed comments from main ideas through technical details. All possible errors are my own and comments are welcome.

†Correspondence: Department of Economics, University of Chicago,Chicago,IL 60637. Tel: 773-401-0382; Email: wangyong@uchicago.edu.

1 Introduction

Economists have come to the consensus that the most fundamental factors accounting for the huge differences in productivity and per capita income acorss countries are the institutions and economic policies.(see North, 1990; Hall and Jones,1999; Prescott, 2002; Acemoglu and Johnson, 2005,etc.). Given that most countries are less developed countries and most people on the earth are living in developing economies, it's of large welfare significance and aslo natural to ask how to make the institutions of the development economies better. In the real world, the last two decades did see a widespread wave of market-orientated economic reforms and institutional transitions in many previous central-planning economies such as China, Russia, and some other East European countries. These institutional changes were largely triggered by the people's growing discontent for the lasting slow or stagnant consumption growth as compared with the sucessful developed economies.To answer the question of how to undertake the reform, it might be necessary to first analyze how the previous "inefficient institutions" came into being since institutions are perhaps the endogenous outcome of some more fundamental social forces and mechanisms.There is a vast amount of literature addressing the various aspects of the institutional formation, economic transitions, and reforms(see Bruno1972; Murphy, Shleifer and Vishny, 1992; Dewatripont,M. and G. Roland, 1992a, 1992b; Lin, 1994, 2002). Unfortunately, even in the theoretical community the debates have never stopped between the advocates of radical reforms within the camp named "Washington Consensus" and the supporters in the camp for the gradual reforms labelled as " Beijing Consensus". It still remains a puzzle why the radical reform strategy advocated by many economists worked well in some south American countries while failing to acheive the expected positive outcomes in some other Eastern European countries. How can China, which adopted a gradual reform strategy, keep growing at an unprecendented high speed for nearly thirty years and there are by far still no signs indicating that this trend will stop in the decades to come? This paper, in contrast with many previous research, will mainly explore the dynamics of a costly insitutional change in a highly stylized transitional model , in which I am trying to characterize how should the central government of a developing country, as a social planner, optimally adjust its "institutional barrier" that affects its growth due to the human capital externality from abroad and how this reform would affect the level and growth rate of GDP.

I study the following simple dynamic planning problem: The domestic human capital accumulates at a constant speed thanks to externality of a representative developed economy's human capital until the human capital gap between the two countries becomes small enough such that the catching-up stops.The welfare-maximizing planner formulates a dynamic adjustment scheme specifying when and how to relax the institutional barrier in order to keep the learning process hence growth continuing. The existence of initial barrier is exogenous to the model. The barrier adjustment is costly and likely to be discontinuous as it entails fixed adjustment cost. I first examine the benchmark case in which any beneficial adjustment is essentially always affordable, and then I go on to the case in which the budget constraint might be binding and explicitly characterize the two different adjustment schemes. I believe that a comparative analysis of these two scenarios can not only help us understand the mechanics of institutional change, especially the simple economic logic of the gradual reforms in countries like China, but also shed some light on the role of foreign aid. international capital market in this process. Our main result is that both once-and -for-all reform and piecemeal reform could be optimal in the sense of welfare maximizing, depending on the initial conditions, especially the GDP gaps and the adjustment cost function, and on the completeness of capital market. The model can also explain why economic growth rates flucatuate along with the institutinal change.

The household plays virtually a very trivial role in my model. This deserves further justifications in addition to mere modelling simplicity. When the institutions are taken as exogenous and static, as is typically assumed in most standard growth literature, the most important decision is the household's time and talent allocations over diffferent activities such as human capital accumulation, research and development ,or physical production(See Romer 1990, Lucas,1989,1993,etc.). This assumption is innocuous when we study those mature market economies but might appear unsatisfactory when dealing with the transitional economies where the institutional change is one essential part of the endogenous economic consequences. In our problem, a central planner model is more appealing than a competitive market model because in reality the central governments of these transitional economies almost all have a far greater administrative power than their counterparts in those developed market economies in terms of shaping and changing the institutional barrier through issuing various policies or directly exercising administrative control. In fact, the central governments in these transitional countries often do have very formal and extensive five-year, ten-year or twenty-year plans to reform the economic institutions, and these plans are indeed implemented by the governments. Institutions affect individuals' incentives and hence their optimal behaviors, thus from a macroeconomic point of view,it seems very natural to first examine the government's institutional adjustment behavior before we analyze the optimal response of the households[1]. So this model will just focus on the Ramsey government's behavior,which seems of first order importance.

Generally speaking, there are two approaches to analyze government's behavior. One is the positive approach, which at least dates back to the "contractarian" analysis by Buchanan (1975, 1987) . This approach emphasizes the concrete political process of the policy making and the recent works include the political economic models of trade policies and special interest group by Grossman and Helpman (2001), the transaction-cost analysis for the policy-making process by Dixit (1996), and a series of endogenous policy papers by Persson and Tabellini (2002), etc. The other approach is normative, which emphasizes ideally what an optimal policy should look like according to some efficiency criterion. Many textbook examples are in this category such as the Ramsey government models in optimal macroeconomic policy and time consistency literature (for instance, Kydland and Prescott, 1977 and Lucas and Stokey, 1983,etc.). This paper belongs to

[1] In the extension to a political economic model, we might need to explicitly consider the voting behavior etc., then the roles played by the household or special interest groups need to be considered explicitly and carefully..

the second category, which simply attempts to explore how a hypothetically benevolent central planner should dynamically adjust the exogenously existing institutional barrier, which is abstracted as a one dimentional quantitative control variable. Of course,in reality the central government could have far more complicated goals and the institutional barriers could have many other dimensions,some of which might be even not adjustable,at least in the short run. However, the analysis here might be a useful benchmark for comparison and a starting point for further treatment on these issues.

This paper is organized as follows: Section 2 describes the model economy; The next two sections discuss the optimal barrier adjustment under certainty in two different scenarios. In Section 3, the adjustment cost is financed as an aid by an outside agency with symmetric information, or, equivalently, the international capital market is perfect in the sense that the developing economy can have sufficient large positive sovereign debt without default. Section 4 tries to characterize the optimal adjustment policy when all the adjustment cost has to be financed by the developing economy itself without aid and borrowing. The relevance of these theoretical results is briefly discussed and some empirical facts are presented in Section 5. In Section 6, I suggest some possible interesting avenues for further research. The last section concludes.

2 Model Economy with Deterministic Externality

In this continuous-time world, the decision maker is the benevolent central government of the less developed country, who wishes to maximize the following total discounted utility of per capita consumption stream over an infinite horizon:

$$\int_0^\infty c(t)e^{-\rho t}dt, \tag{1}$$

where ρ is the positive discount rate. $c(t) \geq 0, \forall t$. The infinite inter-temporal elasticity of substitution implied by the linear utility function makes consumption analysis trivial once the interest rate r [2]and the output stream are known because what matters is only the present discounted value of the total consumptions, which has to be nonnegative in this case. The timing of consumption is irrelevant so long as it respects the budget constraint [3].Each household is identically endowed with one unit of labor at every time point, which is inelastically supplied to produce the consumption good with the following constant return to scale technology:

$$f(h(t)) = h(t),$$

that is, one unit of human capital[4], $h(t)$, combined with one unit of labor, can produce one unit of storable consumption good per period, which can be used to pay the adjustment

[2]I will assume throughout the paper that the interest rate is exogenous and constant over time. So this developing economy can't affect the international captial market.

[3]We will specify the two different budget constraints, under complete capital market and under incomplete capital market, in the next two sections.

[4]It's worth emphaszing that here human capital actually refers to the combination of all the implicit accumulative production factors including techonology in this economy without physical captial.

cost of learning barrier as well. The initial human capital is h_0.So in this model GDP is simply equivalent to human capital, thus later on throughout the paper I will simply focus ouranalysis on the dynamics of human capital.

There is a developed country with the same physical production technology and its human capital stock is denoted by $H(t)$ at time t, which grows exogenously at a constant exponential speed $g_H \geq 0$.$H(0)$ is normalized to unity. Let $x(t) \equiv \frac{h(t)}{H(t)}$ measure the gap in human capital stocks (or equivalently, gap in GDP)at time t between these two countries. So $x(0) = h_0$. Due to the externality of human capital through various channels such as knowledge spillover etc., $h(t)$ will increase, but the effectiveness of this positive externality is affected by the less developed country's institutions such as its openness, protection of intellectual property right, etc. The adjustable barrier variable is denoted by the real function $\delta(t)$. Let δ_0 denote the initial barrier value. A closer economy would have a larger learning barrier,*ceteris paribus*. The deterministic law of motion due to externality has the following form[5]:

$$\frac{dx(t)}{dt} = \begin{cases} \mu, & if \quad x(t) \leq \frac{\eta}{\delta(t)} \\ 0, & otherwise \end{cases}, \tag{2}$$

where μ and η are both positive. That is, the human capital gap shrinks at a constant speed μ until the gap hits the critical boundary $\frac{\eta}{\delta(t)}$,at which point the gap stops shrinking unless the barrier variable $\delta(t)$ is adjusted downward, which is what we mean by institutional change. A higher η implies a longer time for the less developing country to enjoy the catching-up for any given institution barrier.

When the control variable $\delta(t)$ is changed from δ to δ' in a single adjustment, the associated cost is determined by:

$$C(\delta, \delta') = \begin{cases} A\left(\frac{\delta}{\delta'}\right)^{\phi} + B, & if\ \delta \neq \delta' \text{ and } \delta' \geq \eta \\ \infty, & if\ \delta' < \eta \\ 0, & if\ \delta = \delta' \end{cases}, \tag{3}$$

where A and B are both positive, and their sum is the lowest limit of the cost for each adjustment. The parameter $\phi \geq 2$ indicates that the barrier adjustment cost function is convex in the size of adjustment ,$\frac{\delta}{\delta'}$, capturing the fact that too radical adjustment entails disproportionately higher cost such as increasing oppositions from the harmed interest groups and the social instability etc. No adjustment naturally incurs no cost. (3) imposes a lower bound for the admissible barrier variable, η[6]. This is to rule out the possibility that the less developed country can exceed the developed country simply

[5]It is straightforward to generalize this two-value step function to a multiple-value step function in order to approximate any continuous law of motion. Here we adopt this simplest functional form mainly for tractability and simplicity.

[6]For example, one possible functional form to gurantee $\delta_{i+1} < \eta$ is $C(\delta_i, \delta_{i+1}) = \begin{cases} A\left(\frac{\delta_i}{\delta_{i+1}-\eta}\right)^{\phi} + B, & if\ \delta_i \neq \delta_{i+1} \text{ and } \delta_{i+1} > \eta \\ 0, & o/w \end{cases}$, however, this functional form will make computation even more complicated.

by the externality in human capital. Thus $x(t) \leq 1, \forall t$. I allow for upward adjustment capturing the possibility of issuing some new policies which impede externality, but in this model obviously the benevolent government has no incentive to do so hence the relevant adjustment must be downward. The adjustment policy functional space is

$$\Delta \equiv \{\text{real function } \delta(t) : \mathbb{R}_+ \to [\eta, \delta_0] \text{ such that } \delta(0) = \delta_0 > \eta\}.$$

Since any real adjustment entails fixed cost, a typical time path of the optimal adjustment policy $\delta^*(\cdot)$ is a step function rather than a continuous function. So equivalently the decision maker needs to find a bounded and weakly decreasing sequence $\{\delta_i\}_{i=0}^{\infty}$ and the corresponding adjustment time sequence $\{t_i\}_{i=0}^{\infty}$ with δ_0 and $t_0 = 0$ given, where δ_i stand for the value of the barrier variable after the ith adjustment.

Since the institutional adjustment often entails tremendous cost, the budget constraint is very crucial for our analysis of any feasible adjustment plan. In the next two sections, I will specify two different budget constraints depending on whether the capital market is complete or not.

Let's start with the easier baseline case in which the capital market is complete so the decision maker only needs to respect the inter-temporal budget constraint.

3 Adjustment Under Complete Capital Market

Assume that the international capital market has an exogenous interest rate equal to $r = \rho - g_H$. For simplicity, I also assume $g_H = 0$[7],and $\mu > r > 0$.If the international capital market is complete, then the less developed country can , if necessary,borrow capital from the market to finance its institutional adjustment so long as it can and do pay back its debt later. The existence of positive amount of sovereign debt would typically require the strong assumption of full commitment or effective enforcement such as by collateralizing between the two parties of the loan contract. However, Bulow and Rogoff (1989) show that it is a sequential equilibrium that no positive sovereign debt would exist if no collateral is available and even reputation effect still can't prevent default on sovereign debt. So without imposing complete market or full commitment , I will adopt a different but analytically equivalent assumption, namely, let's assume there exists an international agency called "World Bank"(WB) which can provide foreign aid to this developing economy to finance its downward barrier adjustment. WB has sufficient but finite fund and also has access to the international market. The developing country can't have any positive amount of sovereign debt although it can save in the international capital market. However, the foreign aid is conditional in the sense that the benefit of the adjustment has to more than compensate the adjustment cost. Foreign aid is more than a merely hypothetical apparatus for analysis, it is a very important practical and policy

[7]Hence the discount rate ρ is just equal to the interest rate ,r,on the international capital market.The extension to the case of $g_H > 0$ is quite straightforward since the growth rate g_H is exogenous and fixed. However, the consumption behavior becomes a little more complicated since $\rho \neq r$.

issue per se concerning helping poor countries out of "poverty trap". There are many researches on the effectiveness of foreign aid and why foreign aid might be very important or even indispensable for facilitating institutional reforms, see Fischer(1991).Under symmetric information, WB can be equivalently taken as the virtual planner with transferable utilities between itself and the developing economy which receives the aid.

Now the optimization problem is to find an optimal adjustment scheme, $\{\delta_i, t_i\}_{i=1}^{\infty}$, and optimal time path of consumption, $c(t)$, to maximize (1) subject to (2), (3), δ_0 and $t_0 = 0$ are given, $c(t) \geq 0$ for all t, and the following budget constraint:

$$\int_0^{\infty} c(t)e^{-rt}dt \leq \sum_{i=0}^{\infty}[\int_{t_i}^{t_{i+1}} h(t)e^{-rt}dt - C(\delta_i, \delta_{i+1})e^{-rt_{i+1}}], \tag{4}$$

that is, the total present discounted value of consumption has to be no larger than the total present discounted value of output net of all the adjustment cost.

(4) must hold as equality by the strong monotonicity of the utility function and I don't need to pin down the particular optimal time path of consumption for the reason given in the last section. These two facts together with $r = \rho$ enable us to reformulate the optimization problem as following:

The value function for the planner is

$$V(\delta_0, x_0) = \max_{\{\delta_i, t_i\}_{i=1}^{\infty}} \sum_{i=0}^{\infty}[\int_{t_i}^{t_{i+1}} x(t)e^{-\rho t}dt - C(\delta_i, \delta_{i+1})e^{-\rho t_{i+1}}] \tag{5}$$

subject to (2), (3), δ_0 and $t_0 = 0$ are given, and that the associated adjustments must be always affordable, namely:

$$\sum_{i=0}^{\infty}[\int_{t_i}^{t_{i+1}} x(t)e^{-\rho t}dt - C(\delta_i, \delta_{i+1})e^{-\rho t_{i+1}}] \geq 0 \tag{6}$$

Moreover, no adjustment (that is, setting $\delta_i \equiv \delta_0, \forall i$, or equivalently, setting $t_1 = \infty$) is always a feasible choice, in which case (6) holds as a strict inequality, so the feasible adjustment space is not an empty set and the budget constraint (6) can be safely ignored as it is never binding for any optimal solution. Obviously, $V(\delta_0, x_0)$ must be bounded both from above and from below because $x(t) \leq 1 \ \forall t$.

The law of motion for $x(t)$, (2) ,suggests that $x(t)$ might not be always differentiable although continuous, and the cost function given in (3) is path-dependent, so we will have to solve this problem sequentially.We denote the value function with a total of i adjustment opportunities by V_i .Therefore the value function with no adjustment is

$$V_0(\delta_0, x_0) = x_0 \int_0^{T_0} e^{\mu t}e^{-rt}dt + e^{-rT_0} \int_0^{\infty} \frac{\eta}{\delta_0}e^{-rt}dt$$

where the catching-up stops at

$$T_0 = \max\{0, \frac{1}{\mu} \ln \frac{\eta}{\delta_0 x_0}\} \tag{7}$$

Therefore,

$$V_0(\delta_0, x_0) = \begin{cases} \frac{\eta}{\delta_0} \frac{\mu}{r(\mu - r)} \left(\frac{\eta}{\delta_0 x_0}\right)^{\frac{-r}{\mu}} - \frac{x_0}{\mu - r}, & \text{if } \delta_0 x_0 < \eta \\ \frac{\eta}{\delta_0 r}, & \text{if } \delta_0 x_0 \geq \eta \end{cases} \tag{8}$$

From now on let's only focus on the more interesting case in which the initial human capital is sufficiently low:

Assumption 0:

$$x_0 < \eta / \delta_0 \tag{A0}$$

If there is only one opportunity to exercise a control, then it can be exercised either weakly before the learning constraint is binding , in which case we have:

$$G_1(\delta_0, x_0) = \max_{T \leq T_0, \delta_1} \int_0^T x_0 e^{\mu t} e^{-rt} dt + e^{-rT} \left[V_0(\delta_1, x_0 e^{\mu T}) - C(\delta_0, \delta_1)\right], \tag{9}$$

or after the learning constraint is binding, in which case we have:

$$F_1(\delta_0, x_0) = \max_{T_0 \leq T_1, \delta_1} \int_0^{T_0} x_0 e^{\mu_0 t} e^{-rt} dt + \int_{T_0}^{T_1} \frac{\eta}{\delta_0} e^{-rt} dt + e^{-rT_1} \left[V_0(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1)\right]. \tag{10}$$

Note

$$G_1(\delta_0, x_0) = V_0(\delta_0, x_0), \text{if } \delta_1^* = \delta_0;$$

and

$$F_1(\delta_0, x_0) = V_0(\delta_0, x_0), if\ T_1^* = \infty \text{ in } (10)$$

The value function is thus

$$V_1(\delta_0, x_0) = \max\{G_1(\delta_0, x_0), F_1(\delta_0, x_0)\}$$

Lemma 1 *Under assumption A0,*

$$V_1(\delta_0, x_0) = F_1(\delta_0, x_0), \forall (\delta_0, x_0)$$

Proof. By contradiction, suppose $\exists$ an optimal adjustment time $T^* \in (0, T_0)$ and a real adjustment is made so that $\delta_1^* \neq \delta_0$. Substituting (8) into (9), we can easily prove

$$\frac{\partial}{\partial T} \left\{ \int_0^T x_0 e^{\mu t} e^{-rt} dt + e^{-rT} \left[V_0(\delta_1, x_0 e^{\mu T}) - C(\delta_0, \delta_1)\right] \right\} > 0, \forall \delta_1 \neq \delta_0, \forall T \in (0, T_0]$$

Moreover, any adjustment affordable at T^* must be affordable at T_0 . If it's optimal not to exercise the adjustment, $F_1(\delta_0, x_0) = V_0(\delta_0, x_0)$, where $T^* = \infty$. Contradiction. ■

The intuition behind this lemma is the following: For any real adjustment made strictly before the learning constraint becomes binding (i.e. T_0), the net value can be strictly increased if the same amount of adjustment is made at T_0 because the gross benefit of any such adjustment is independent of the adjustment time but the payment of the same adjustment cost is now delayed and hence the present discounted value of the cost is decreased. This proof simply shows that

$$G_1(\delta_0, x_0) = \max\left\{V_0(\delta_0, x_0), \max_{\delta_1} \int_0^{T_0} x_0 e^{\mu t} e^{-rt} dt + e^{-rT_0}\left[V_0(\delta_1, x_0 e^{\mu T_0}) - C(\delta_0, \delta_1)\right]\right\} \leq F_1(\delta_0, x_0),$$

for any (δ_0, x_0) satisfying A0.

This lemma allows us to simply focus on the real adjusment made only weakly after the learning barrier becomes binding.

Assumption 1:

$$A\phi r < \frac{\eta}{\delta_0} \leq (A\phi r)^{\frac{1}{\phi + \frac{r}{\mu}}} \tag{A1}$$

The first strict inequality of A1 ensures that the adjustment, if made, must be downward. The right weak inequality guarantees that the initial barrier is sufficiently high such that at least the first adjustment will be interior as will be explained in the proof of Proposition 3 below.

Definition 2

$$\widetilde{B}(y) \equiv A\left[\frac{A\phi r y}{\eta}\right]^{\frac{-\phi}{\phi + \frac{r}{\mu} - 1}}\left(\frac{\phi\mu}{\mu - r} - 1\right) - \frac{\eta\mu}{r(\mu - r)y}$$

Proposition 3 *Under assumption A1, if the fixed adjustment cost is sufficiently small such that* $B < \widetilde{B}\ (\delta_0)$, *then WB will provide aid , an optimal downward barrier adjustment will be made exactly when the learning barrier first becomes binding, and the total welfare will be strictly increased.*

Proof. *By Theorem 1 and the functional form of* V_0 *in (8), we have the following two first order conditions with respect to* T_1 *and* δ_1 :

$$\frac{\partial F_1(\delta_0, x_0)}{\partial T_1} = re^{-rT_1}[V_0(\delta_0, \frac{\eta}{\delta_0}) + C(\delta_0, \delta_1) - V_0(\delta_1, \frac{\eta}{\delta_0})]$$

$$\frac{\partial F_1(\delta_0, x_0)}{\partial \delta_1} = 0 \Rightarrow \delta_1^* = \theta(\delta_0)\delta_0 \in [\eta, \delta_0) \text{ guaranteed by A1}$$

where

$$\theta(\delta_0) \equiv \left[\frac{A\phi r\delta_0}{\eta}\right]^{\frac{1}{\phi + \frac{r}{\mu} - 1}}, \tag{11}$$

Assumption A1 guarantees that $\delta_1^* \in [\eta, \delta_0)$. The condition $B < \widetilde{B}\ (\delta_0)$ insures

$$V_0(\delta_0, \frac{\eta}{\delta_0}) + C(\delta_0, \delta_1^*) - V_0(\delta_1^*, \frac{\eta}{\delta_0}) > 0$$

and hence

$$T_1^* = T_0.$$

The second order condition can be verified to hold. ■

Using the results in the above proof , we obtain:

$$V_1(\delta_0, x_0) = \frac{\mu\eta}{r(\mu - r)} \left(\frac{\eta}{\delta_0 x_0}\right)^{\frac{-r}{\mu}} \theta(\delta_0)^{\frac{r}{\mu}-1}\delta_0^{-1} - \frac{x_0}{\mu - r} - \left(\frac{\eta}{\delta_0 x_0}\right)^{\frac{-r}{\mu}} (A\theta(\delta_0)^{-\phi} + B), \quad (12)$$

where $\theta(\delta_0)$ is given by (11).Recall the present discounted value of the adjustment cost ,$\left(\frac{\eta}{\delta_0 x_0}\right)^{\frac{-r}{\mu}} (A\theta^{-\phi} + B)$,is exclusively financed by WB, so the developing country's welfare equals

$$\frac{\mu\eta}{r(\mu - r)} \left(\frac{\eta}{\delta_0 x_0}\right)^{\frac{-r}{\mu}} \theta(\delta_0)^{\frac{r}{\mu}-1}\delta_0^{-1} - \frac{x_0}{\mu - r},$$

which is strictly larger than $V_0(\delta_0, x_0)$,the welfare level with no adjustment.

Now if the developing country has two adjustment opportunities that will be financed by WB conditional on that the adjustment must be Pareto-improving in the sense that $V_2(\delta_0, x_0) > V_1(\delta_0, x_0)$, the value function becomes

$$V_2(\delta_0, x_0) = \max\{G_2(\delta_0, x_0), F_2(\delta_0, x_0)\},$$

where

$$G_2(\delta_0, x_0) = \max_{T_1 \le T_0, \delta_1} \int_0^{T_1} x_0 e^{\mu t} e^{-rt} dt + e^{-rT_1} \left[V_1(\delta_1, x_0 e^{\mu T_1}) - C(\delta_0, \delta_1)\right],$$

and

$$F_2(\delta_0, x_0) = \max_{T_0 \le T_1, \delta_1} \int_0^{T_0} x_0 e^{\mu_0 t} e^{-rt} dt + \int_{T_0}^{T_1} \frac{\eta}{\delta_0} e^{-rt} dt + e^{-rT_1} \left[V_1(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1)\right].$$

The welfare level with two adjustment opportunities is obviously no lower than that with only one adjustment since the option can be simply waived freely. The first order condition with respect to δ_1 gives the following nonlinear equation:

$$B\frac{r}{\mu}\delta_1^{\frac{r}{\mu}+\phi} + k\delta_1^{\frac{r}{\mu}+\phi-\frac{\phi}{\frac{r}{\mu}+\phi-1}} - A\delta_0^{\phi+\frac{r}{\mu}}\phi = 0, \quad (13)$$

where

$$k = \frac{\eta(\phi\mu + r)}{r\phi\mu} \left(\frac{Ar\phi}{\eta}\right)^{\frac{\frac{r}{\mu}-1}{\frac{r}{\mu}+\phi-1}}.$$

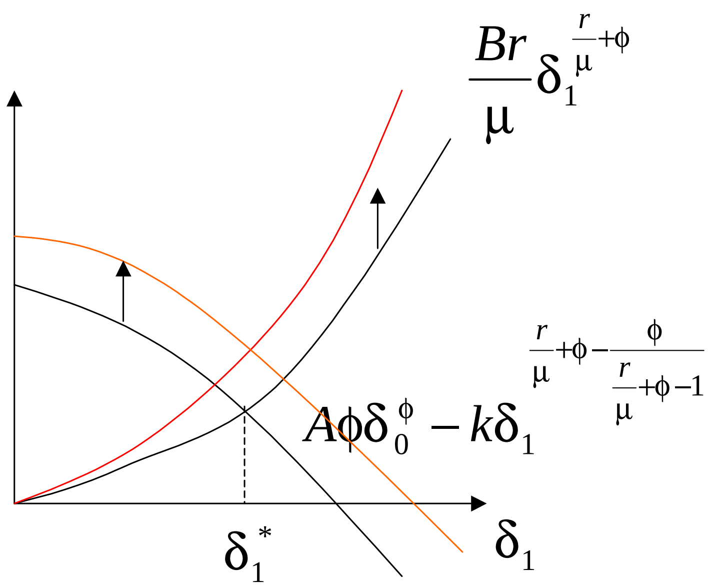

Figure 1: Optimal Adjustment Size

Although no closed-form solution for the optimal adjustment size can be obtained for this case, Figure 1 shows that the solution exists and it is unique.

From the first order condition

$$\frac{\partial F_2(\delta_0, x_0)}{\partial T_1} = re^{-rT_1}[V_0(\delta_0, \frac{\eta}{\delta_0}) + C(\delta_0, \delta_1) - V_1(\delta_1, \frac{\eta}{\delta_0})],$$

we can conclude $T_1^* = T_0$ for $F_2(\delta_0, x_0)$ if and only if $\Delta(\delta_0, \delta_1^*) \geq 0$, where

$$\Delta(\delta_0, \delta_1^*) = (\delta_1^*)^{\frac{r}{\mu}-\frac{\phi}{\frac{r}{\mu}+\phi-1}} \left(\frac{Ar\phi}{\eta}\right)^{\frac{\frac{r}{\mu}-1}{\frac{r}{\mu}+\phi-1}} \frac{\eta}{r\phi}\left(\frac{\phi\mu}{\mu-r}-1\right)\delta_0^{-\frac{r}{\mu}} - \frac{\eta\mu}{r(\mu-r)\delta_0} - B - B\left(\frac{\delta_1^*}{\delta_0}\right)^{\frac{r}{\mu}} - A\delta_0^{\phi}\delta_1^{*-\phi},$$

where δ_1^* is the unique solution to equation (13). Simple comparative statics show that an increase in the fixed adjustment cost B will move δ_1^* leftward, this is because costing-saving will make adjustment less frequent but the size for each adjustment larger. In contrast, higher A results in higher δ_1^* because the variable adjustment cost parameter A affects the marginal adjustment cost. The higher the initial barrier, the more modest the first target of barrier adjustment. When $\Delta(\delta_0, \delta_1^*) < 0$, at least one control option will be waived:

$$V_2(\delta_0, x_0) = V_1(\delta_0, x_0).$$

Suppose there are N options to exercise a control, where $N \geq 2$, then

$$V_N(\delta_0, x_0) = \max\{G_N(\delta_0, x_0), F_N(\delta_0, x_0)\},$$

where

$$G_N(\delta_0, x_0) = \max_{T_1 \le T_0, \delta_1} \int_0^{T_1} x_0 e^{\mu t} e^{-rt} dt + e^{-rT_1} \left[V_{N-1}(\delta_1, x_0 e^{\mu T_1}) - C(\delta_0, \delta_1) \right],$$

and

$$F_N(\delta_0, x_0) = \max_{T_0 \le T_1, \delta_1} e^{-rT_1} \left[V_{N-1}(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1) - V_0(\delta_0, \frac{\eta}{\delta_0}) \right] + \frac{x_0 \mu}{\mu - r} \left(\frac{\eta}{\delta_0 x_0} \right)^{1-\frac{r}{\mu}} - \frac{x_0}{\mu - r}.$$

Proposition 4 *Suppose there are a total of $N \geq 2$ opportunities to make adjustments. If at least one real adjustment has been made for the first $N-1$ options, then under Assumption A1, the last adjustment opportunity won't be waived if and only if*

$$\delta_{N-1}^* \geq \eta \widetilde{y}(\delta_{N-1}^*),$$

whereδ_{N-1}^* is the value of the barrier variable after the first $N-1$ adjustments and $\widetilde{y}(\delta_{N-1}^*)$ is the unique root of $\Omega(y, \delta_{N-1}^*)$,which is defined as following:

$$\Omega(y, \delta_{N-1}^*) \equiv \frac{\mu \eta}{\delta_{N-1}^* r(\mu - r)} \left[y^{1-\frac{r}{\mu}} - 1 \right] - \left[A y^{\phi} + B \right]$$

Moreover, the last control must be made precisely when the learning barrier just becomes binding.

Proof.

$$V_1(\delta_{N-1}^*, x_{N-1}^*) = \max\{G_1(\delta_{N-1}^*, x_{N-1}^*), F_1(\delta_{N-1}^*, x_{N-1}^*)\},$$

By downward adjustment,

$$\delta_{N-1}^* \leq \delta_{N-2}^* \leq \delta_{N-3}^* \cdots \leq \delta_0,$$

since at least one inequality is strict, by Assumption 1, we have

$$A\phi r \delta_{N-1}^* < \eta$$

and

$$\widetilde{B}(\delta_{N-1}^*) \equiv A \left[\frac{A\phi r \delta_{N-1}^*}{\eta} \right]^{\frac{-\phi}{\phi + \frac{r}{\mu} - 1}} \left(\frac{\phi \mu}{\mu - r} - 1 \right) - \frac{\eta \mu}{r(\mu - r)\delta_{N-1}^*} > \widetilde{B}(\delta_0),$$

since $\widetilde{B}'(\cdot) < 0$ under assumption A1. Therefore $B < \widetilde{B}(\delta_{N-1})$.Now let $y = \frac{\delta_{N-1}^*}{\delta_N}$, then

$$\Omega(\frac{\delta_{N-1}^*}{\delta_N}, \delta_{N-1}^*) = V_0(\delta_N, \frac{\eta}{\delta_{N-1}^*}) - C(\delta_{N-1}^*, \delta_N) - V_0(\delta_{N-1}^*, \frac{\eta}{\delta_{N-1}^*})$$

so we must have $\frac{\delta_{N-1}^*}{\eta} \geq \widetilde{y}(\delta_{N-1}^*)$ in order to make it worthwhile to adjust δ_N to the lower boundary η. ■

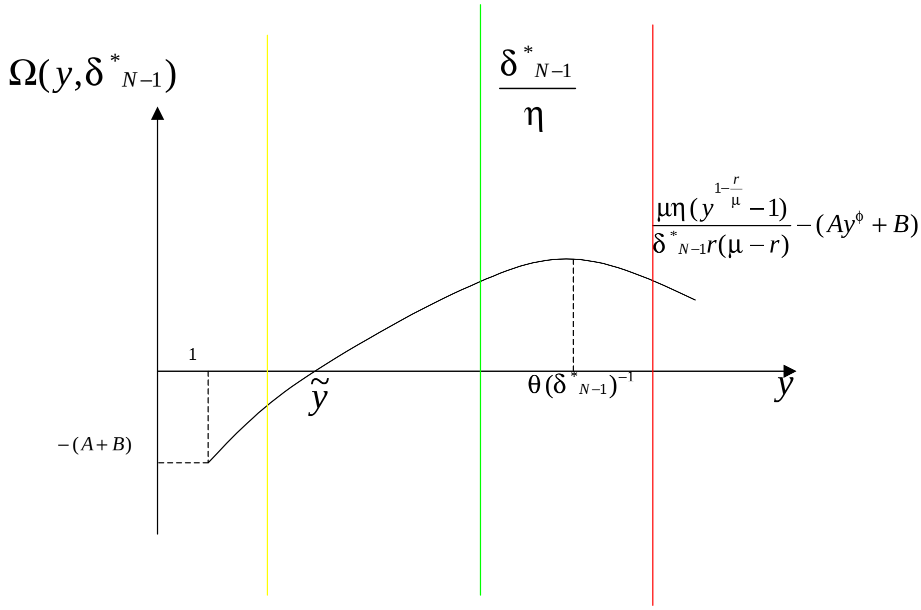

Figure 2: The Last Adjustment with Multiple Options

This proposition can be illustrated in the following figure: Suppose the optimal δ^*_{N-1} is so small that $\frac{\delta^*_{N-1}}{\eta} < \widetilde{y}(\delta^*_{N-1})$ (see the left vertical line), then $\Omega(\frac{\delta^*_{N-1}}{\eta}, \delta^*_{N-1}) < 0$, hence it's optimal to make no adjustment for the last option. If $\widetilde{y}(\delta^*_{N-1}) < \frac{\delta^*_{N-1}}{\eta} \leq \theta(\delta^*_{N-1})^{-1}$, then the last adjustment will be made and ultimately the developing country will have the same human capital stock as the developed country. If $\frac{\delta^*_{N-1}}{\eta} > \theta(\delta^*_{N-1})^{-1}$, then the last adjustment will be made and the ultimate barrier variable will be equal to $\theta(\delta^*_{N-1})\delta^*_{N-1}$.The developing country can never catch up with the developed country.

Conjecture 5 *The optimal number of adjustments is finite under assumption A1 .*

The difficulty lies in the fact that the closed-form policy functions for more than two options can not be obtained, hence I don't know whether or not the optimal adjustment is quick enough so that the critical value $\eta\,(A\phi r)^{\frac{-1}{\phi+\frac{r}{\mu}}}$ is reached within a finite period. Numerical method could be helpful for verifying the correctness of this conjecture.

In this section, given that WB will provide sufficient foreign aid to finance any Pareto-improving adjustment, the developing economy has no incentive to finance its own adjustment even if it can, because under symmetric information, the optimal adjustment characterized in the last section is unique and indeed Pareto optimal, therefore any of its own adjustment fund will crowd out the foreign aid one-to-one and also decrease its own consumption and hence reduce its total discounted utility.Furthermore, the above analysis suggests that WB doesn't have to monitor the implementation of the barrier adjustment all the time because learning externality is deterministic and it can simply calculate the total present discounted value of the adjustment cost and give it to the developing economy as unconditional foreign aid at date zero(or simply right before the first adjustment is made). This won't cause any moral hazard problem since it won't distort the developing economy's goal function and it can guarantee all the optimal adjustment is affordable by this aid. It should be clear that this amount of foreign aid is more than the necessary minimum to guarantee the feasibility of all the welfare-improving adjustment because at least part of the cost can be covered by the developing economy itself.

4 Adjustment Under Incomplete Capital Market

Now suppose we have the imperfect capital market case without foreign aid, so any barrier adjustment has to be financed by its own saving. I still maintain the assumption that $\mu > r = \rho > 0$ and $g_H = 0$ as in the last section. The optimization problem for the central Ramsey government of the less developed country now becomes seeking an optimal adjustment scheme, $\{\delta_i, t_i\}_{i=1}^{\infty}$, and optimal time path of consumption, $c(t)$, to maximize (1) subject to (2), (3),δ_0 and $t_0 = 0$ are given, $c(t) \geq 0$ for all t, and also subject to the following budget constraint:

$$0 \leq S(t) = e^{rt}\left[\sum_{i=0}^{I}\left[\int_{t_i}^{t_{i+1}}(h(s)-c(s))e^{-rs}ds - C(\delta_i,\delta_{i+1})e^{-rt_{i+1}}\right] + \int_{t_i+1}^{t}(h(s)-c(s))e^{-rs}ds\right], \forall t \tag{14}$$

where $I \equiv \max\{i\,|t_{i+1} \leq t\}$.So it says that the consumption flow at any time t has to be fully supported by its own saving up to t, $S(t)$[8],which equals the total output net of all the adjustment cost and consumption up to t measured at the time t value.

Since the infinite inter-temporal elasticity of substitution in consumption can always enable the planner to postpone any consumption to an arbitrarily far but finite future time point[9], technically we can still simplify our consumption analysis by just focusing on the total present discounted value of consumption. So the goal is still to maximize the total present discounted value of consumption same as in (5) subject to (2), (3), δ_0 and $t_0 = 0$ are given, and subject to the real budget constraint obtained from (14), which now becomes[10]

$$\int_{0}^{t_{i+1}} x(s)e^{-rs}ds - \sum_{j=0}^{i}[C(\delta_j,\delta_{j+1})e^{-rt_{j+1}}] \geq 0, \forall i = 0, 1, 2, ..., \tag{15}$$

[8] If some actual adjustment is made at time t, then $S(t)$ jumps down at t. To save notation, I will always define the discountinuous function $S(t)$ as right continuous, so suppose some actual adjustment is made at time t,then $S(t)$ is the current time t value of the saving AFTER the adjustment cost is paid.

[9] This is useful because if there is finite number of adjustments, then all the consumption can be postponed until after all the adjustments have been made, thus we can simply ignore the consumption problem when analyzing barrier adjustment.

[10] By assumption, we have $x(t) = h(t), \forall t$. I will use $x(t)$ just for the purpose of consistency if we later relax the assumption $g_H = 0$.

where I simply let consumption always equal to zero before all the adjustment is over, that is, $c(s) = 0, \forall s < T^{**}$, where T^{**} is the time of the last adjustment. Even if T^{**} is infinite, (15) is still the valid budget constraint because we know that the total present discounted consumption(that is,value function (5)) is strictly positive and bounded from above for the same reason in the last section. So there must be some time point after which positive consumption is feasible so (15) won't be binding after that and hence can be safely ignored. In the appendix, I give an alternative way of formulating this problem using the usual methods in option theory and show why this problem is not standard in the sense that there is no fixed point in the relevant functional space for the related Bellman Equations.

We first need to check whether the adjustment cost obtained in the last section is affordable by the developing economy itself at each of the optimal adjustment times without any outside help. If so, the foreign aid is not necessary. If not, we need to analyze whether there are any sub-optimal choices under the financial constraint.

As pointed out earlier, the value functions and policy functions with more than two options are hard to obtain analytically , primarily because there is no fixed point in the relevant functional space that can be obtained in the related Bellman Equations, especially if the number of options to adjustment is finite. More complicatedly, both the number and the timing of the optimal adjustment are endogenous. So let's mainly focus on the simplest case in which there is only one option to exercise a control,which I call "Shock Therapy".

Lemma 6 *Under Assumption A1 and $B < \widetilde{B}\ (\delta_0)$, the optimal adjustment size of "Shock Therapy" given in the Proposition 3 can be fully financed by the developing economy itself independently if and only if $Q(x_0) \geqslant 0$,where*

$$Q(x_0) \equiv \frac{x_0}{(\mu - r)}\left[\frac{\eta}{\delta_0 x_0} - \left(\frac{\eta}{\delta_0 x_0}\right)^{\frac{r}{\mu}}\right] - (A\left[\frac{A\phi r\delta_0}{\eta}\right]^{\frac{-\phi}{\phi+\frac{r}{\mu}-1}} + B).$$

Proof. *Proposition 3 tells us that if the fixed adjustment cost $B < \widetilde{B}\ (\delta_0)$, then under assumption A1, the adjustment will be indeed made at the timeT_0 given in (7).The current value of the adjustment cost at T_0 is*

$$C(\delta_0, \delta_1) = (A\theta(\delta_0)^{-\phi} + B),$$

where $\theta(\delta_0)$ is still given by (11).The current value of the total output up to T_0 at time T_0 is

$$e^{rT_0}\int_0^{T_0} x_0 e^{\mu t}e^{-rt}dt = \frac{x_0}{(\mu - r)}\left[\frac{\eta}{\delta_0 x_0} - \left(\frac{\eta}{\delta_0 x_0}\right)^{\frac{r}{\mu}}\right]$$

and $Q(x_0)$ is simply the difference between these two. ■

Assumption 2:

$$A\left[\frac{A\phi r\delta_0}{\eta}\right]^{\frac{-\phi}{\phi+\frac{r}{\mu}-1}} + B < \frac{\eta}{(\mu - r)\delta_0} \tag{A2}$$

Obviously, $Q(x_0) < 0$ if A2 is violated, hence the optimal adjustment cost can never been fully financed without foreign help. However, with assumption A2, whether self-financing is enough becomes indeterminate, depending on the initial condition x_0(which is equal to initial human capital stock h_0) . This is the more interesting case ,which I will focus on.

Proposition 7 *Under assumptions A1,A2 and $B < \widetilde{B}\ (\delta_0)$, there exists a critical value x^* such that the cost of the optimal adjustment plan obtained in Proposition 3 (We will call it "World Bank plan " , or WB plan, form now on) can be fully financed by the developing economy itself through saving in the international capital market if and only if the initial human capital stock $x_0 \leq x^*$ and the WB plan will indeed be implemented.*

Proof. *The implementation of this WB plan is established by Proposition 3 if sufficient fund can be found to finance it. It's straightforward to check that $x_0 > 0$ implies $Q'(x_0) < 0$. We can obtain the unique root ,x^*,of the equation $Q(x_0) = 0$:*

$$x^* = \left\{ \eta^{\frac{-r}{\mu}} \left[\eta - (\mu - r)(A \left[\frac{A\phi r \delta_0}{\eta} \right]^{\frac{-\phi}{\phi + \frac{r}{\mu} - 1}} + B) \right] \right\}^{\frac{\mu}{\mu - r}} < \eta,$$

where the second inequality comes from assumption A2. This is consistent with the case

$$\delta_0 x_0 < \eta,$$

which I assume throughout the paper. Therefore,

$$Q(x_0) > 0 \text{ iff } x_0 < x^*.$$

Lemma 10 can be called for to establish the financing ability argument. ■

This result might appear surprising because the financing ability decreases with the initial endowment of the human capital!! This apparent paradox can be resolved as follows: Proposition 3 tells us that the optimal adjustment size won't depend on the initial human capital if there is only one chance to make an adjustment. However, the optimal timing of the adjustment does depend on the initial condition:(7) suggests that the higher the initial human capital, the sooner the optimal adjustment comes, hence the present discounted value of the total adjustment cost will be higher if the initial human capital is higher. Or put differently, the adjustment time comes too soon before enough fund has been accumulated. The proof of Proposition 3 suggests that this timing effect dominates the endowment effect in human capital.

Now if $x_0 > x^*$ so that the ideal adjustment characterized in Proposition 3 is not feasible, how does the developing economy modify its plan under the financial constraint? The value function becomes

$$V_1^{(self)}(\delta_0, x_0) = \max\{G^{(self)}(\delta_0, x_0), F^{(self)}(\delta_0, x_0)\}, \tag{16}$$

where

$$G^{(self)}(\delta_0, x_0) = \max_{T_1 \le T_0, \delta_1} \int_0^{T_1} x_0 e^{\mu t} e^{-rt} dt + e^{-rT_1} \left[V_0(\delta_1, x_0 e^{\mu T_1}) - C(\delta_0, \delta_1) \right] \tag{17}$$

subject to

$$\int_0^{T_1} x_0 e^{\mu t} e^{-rt} dt \ge e^{-rT_1} C(\delta_0, \delta_1) \tag{18}$$

and

$$F^{(self)}(\delta_0, x_0) = \max_{T_0 \le T_1, \delta_1} \int_0^{T_0} x_0 e^{\mu t} e^{-rt} dt + \int_{T_0}^{T_1} \frac{\eta}{\delta_0} e^{-rt} dt + e^{-rT_1} \left[V_0(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1) \right] \tag{19}$$

subject to

$$\int_0^{T_0} x_0 e^{\mu t} e^{-rt} dt + \int_{T_0}^{T_1} \frac{\eta}{\delta_0} e^{-rt} dt \ge e^{-rT_1} C(\delta_0, \delta_1), \tag{20}$$

and as before,I will focus on the case when $\eta > x_0$.

Theorem 1 can be easily adapted to this Shock Therapy analysis with no foreign aid, and it implies $T_1^* \notin (0, T_0)$. That is, if there is only one option to exercise a control and if some adjustment is indeed made(so $0 < T_1^* < \infty$), then it will never be optimal to make the adjustment before the learning constraint is binding. Let's put this important result formally:

Lemma 8 $V_1^{(self)}(\delta_0, x_0) = F^{(self)}(\delta_0, x_0)$.

Assumption 3:

$$\frac{x_0}{(\mu - r)} \left[\frac{\eta}{\delta_0 x_0} - \left(\frac{\eta}{\delta_0 x_0} \right)^{\frac{r}{\mu}} \right] > A + B \tag{A3}$$

Suppose that adjustment is made at time $T_1^* = T_0$,and as before,I will focus on the case when $\eta > x_0$ .Let $\widetilde{\delta}_1$ denotes the largest adjustment target affordable at time T_0, which is determined by the following equation:

$$C(\delta_0, \widetilde{\delta}_1) = \frac{x_0}{(\mu - r)} \left[\frac{\eta}{\delta_0 x_0} - \left(\frac{\eta}{\delta_0 x_0} \right)^{\frac{r}{\mu}} \right],$$

which under assumption A3 yields,

$$\widetilde{\delta}_1 = \delta_0 \psi(\delta_0),$$

where

$$\psi(\delta_0) \equiv \left[\frac{\frac{x_0}{(\mu - r)} \left[\frac{\eta}{\delta_0 x_0} - \left(\frac{\eta}{\delta_0 x_0} \right)^{\frac{r}{\mu}} \right] - B}{A} \right]^{-\frac{1}{\phi}} < 1.$$

and $x_0 > x^*$ implies $\psi(\delta_0) > \theta(\delta_0)$ as the adjustment is not sufficient. The value function is equal to

$$V_1^{(self)}(\delta_0, x_0) = \int_0^{T_0} x_0 e^{\mu t} e^{-rt} dt + e^{-rT_0} \left[V_0(\widetilde{\delta}_1, \frac{\eta}{\delta_0}) - C(\delta_0, \widetilde{\delta}_1) \right] = e^{-rT_0} V_0(\widetilde{\delta}_1, \frac{\eta}{\delta_0}),$$

where $V_0(\cdot, \cdot)$ is given by (8).

Proposition 9 *Suppose $\eta > x_0 > x^*$. When no outside financial help is available and there is only one option to exercise a control, then under assumptions A1,A2, A3, and $B < \widetilde{B}\ (\delta_0)$, it is never optimal to make no adjustment or make adjustment after time $\widetilde{T}_1^*$,where $\widetilde{T}_1^*$is the time at which the adjustment size specified in WB plan first becomes affordable. In particular, there exists a critical value,$\widetilde{y}(\delta_0) \in (1, \theta(\delta_0)^{-1})$. If $\psi(\delta_0)^{-1} < \widetilde{y}(\delta_0)$, then some adjustment will indeed be implemented strictly after T_0.If $\psi(\delta_0)^{-1} > \widetilde{y}(\delta_0)$, then an optimal adjustment must be made at certain time point $T_1^* \in [T_0, \widetilde{T}_1^*]$, and the budget constraint (20) must be always binding.*

Proof. *Note (19) can be rewritten as*

$$\begin{aligned} F^{(self)}(\delta_0, x_0) = & \max_{T_0 \le T_1, \delta_1} V_0(\delta_0, x_0) + \\ & e^{-rT_1} \left[V_0(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1) - V_0(\delta_0, \frac{\eta}{\delta_0}) \right] \end{aligned}$$

subject to (20). Let $y \equiv \frac{\delta_0}{\delta_1}$,

$$\Omega(y, \delta_0) = V_0(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1) - V_0(\delta_0, \frac{\eta}{\delta_0}).$$

Obviously adjustment will be made only if $\Omega(y, \delta_0) > 0$. Please see Figure 3. The proof of proposition 3 indicates $\Omega(y, \delta_0)$ is maximized at $y = \theta(\delta_0)^{-1}$, and the option value is simply $\Omega(y, \delta_0) > 0$ when $B < \widetilde{B}\ (\delta_0)$. Since $\Omega^+(1, \delta_0) = -(A + B) < 0$ and $\Omega(y, \delta_0)$ is strictly increasing in y on $(1, \theta(\delta_0)^{-1})$, by intermediate value theorem, there exists a unique root of $\Omega(y, \delta_0)$,denoted as $\widetilde{y}(\delta_0)$. When $\psi(\delta_0)^{-1} < \widetilde{y}(\delta_0)$, no affordable adjustment is profitable enough to compensate the adjustment cost at T_0. Nevertheless, it's easy to establish that any adjustment will become affordable as saving lasts for a sufficiently long period. Given the positive option value, the WB plan will be implemented at a finite time $\widetilde{T}_1^*$ determined by

$$\int_0^{T_0} x_0 e^{\mu_0 t} e^{-rt} dt + \int_{T_0}^{T_1^*} \frac{\eta}{\delta_0} e^{-rt} dt = e^{-rT_1^*} C(\delta_0, \theta(\delta_0)\delta_0).$$

If $\psi(\delta_0)^{-1} > \widetilde{y}(\delta_0)$, then at T_0, the total value of the output up to that time is large enough to support a profitable adjustment. The monotonicity of $G(\cdot)$ on the relevant interval guarantees that if adjustment is made at T_0 , it must be adjusted to $\psi(\delta_0)\delta_0$,the largest adjustment allowed by the budget constraint. It's also easy to establish that no adjustment would be later than $\widetilde{T}_1^*$. ■

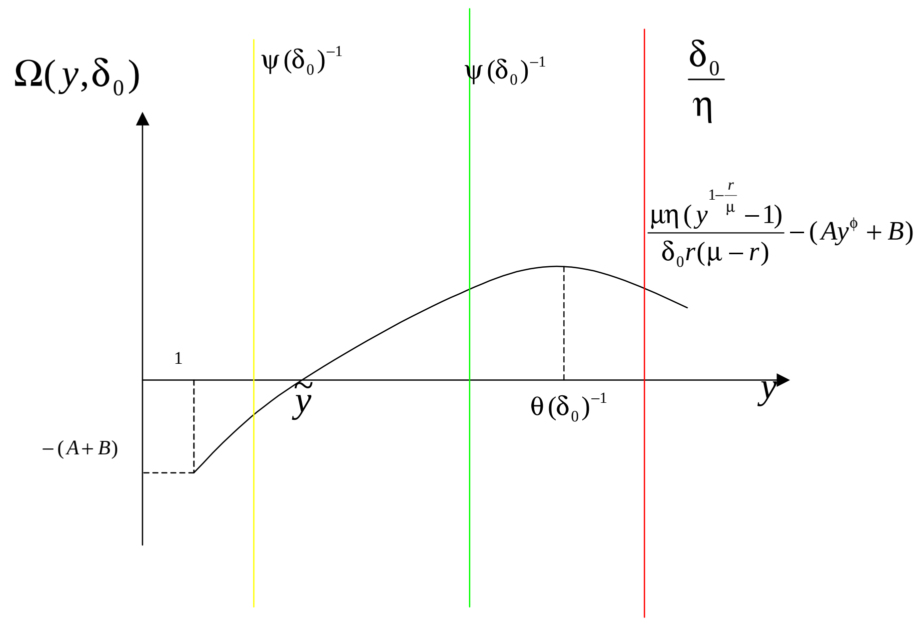

Figure 3: The Optimal Adjustment in the "Shock Therapy" without Foreign Aid

In fact, the optimal δ_1^* is the root of the following nonlinear equation

$$\frac{\frac{\partial V_0(\delta_1,x_0)}{\partial \delta_1}}{V_0(\delta_1,x_0)}=\frac{\frac{\partial[C(\delta_0,\delta_1)+V_0(\delta_0,\frac{\eta}{\delta_0}]}{\partial\delta_1}}{C(\delta_0,\delta_1)+V_0(\delta_0,\frac{\eta}{\delta_0})}, \tag{21}$$

which says the optimal target barrier should be such that the marginal percentage increase in the value due to the barrier adjustment is equal to the marginal percentage increase in the total cost which comprises the direct adjustment cost $C(\delta_0,\delta_1)$ and the foregone utility level after the learning barrier is binding. The reason why we have equality of marginal percentage benefit and cost rather than the usual equality of marginal benefit and cost is that our adjustment cost function given by (3) is essentially determined by the percentage adjustment size rather than by the adjustment size in the absolute levels. (21) can be equivalently written as

$$M\delta_1^{\frac{r}{\mu}+\phi-1}+H\delta_1^{\frac{r}{\mu}-1}+\frac{A\phi\delta_0^{\phi}x_0}{\mu-r}=0.$$

where

$$M=(B+\frac{\eta}{r\delta_0})\frac{\eta}{r}\left(\frac{\eta}{x_0}\right),\ H=A\delta_0^{\phi}\frac{\eta}{r}\left(\frac{\eta}{x_0}\right)^{-\frac{r}{\mu}}(1-\frac{\phi\mu}{\mu-r}).$$

After we obtain the solution δ_1^* from the above equation, the optimal adjustment time is simply given by

$$T_1^*=\frac{1}{r}\ln\frac{C(\delta_0,\delta_1^*)+V_0(\delta_0,\frac{\eta}{\delta_0})}{V_0(\delta_0,x_0)}.$$

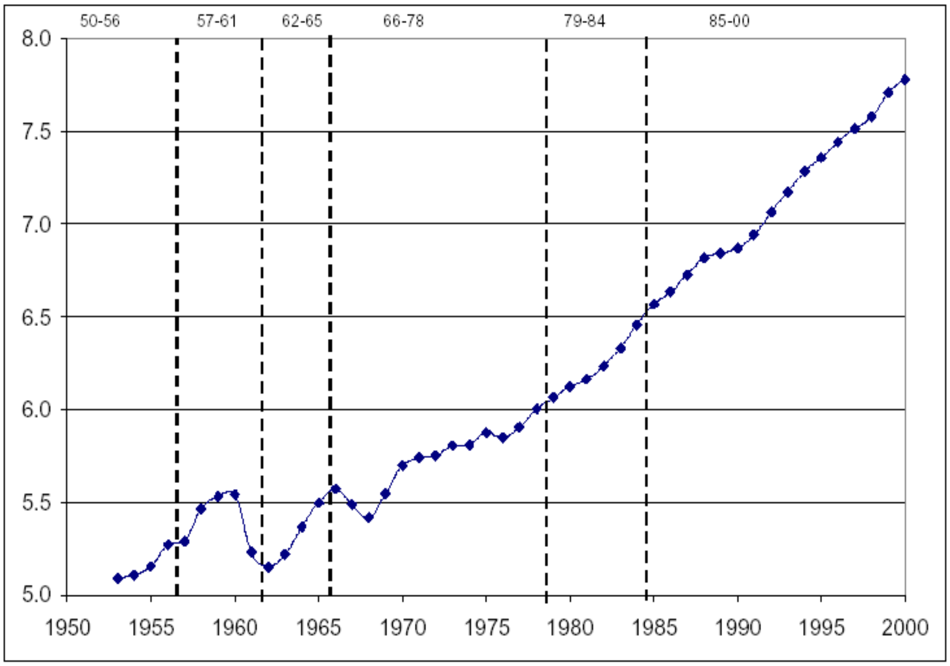

Fig. 1. *Per Capita GDP (in logs) in Constant 1980 Prices*

Figure 4: Source:Ravi Kanbur and Xiaobo Zhang, 2003

As we have seen in the last section, more than two options of adjustment appears very unwieldy at the parametric level even without worrying about the budget constraint. It's natural to expect that the analysis will be even more complicated when the budget constraint is endogenous in this section.

5 Implications of the Model and Some Empirical Facts (tentative)

The above theoretical model might be helpful when we are thinking about the economic development issues in those less developed countries such as China, which has more than one fifth of the total population on this planet and is the largest developing and transitional economy in the world. Figure 1 plots China's real log level of GDP in the last half century.

After the People's Republic of China was founded in 1949, it suffered two major economic recessions. One is the so called 1959-1962 Great Leap Forward Movement and great famine due to the people's loss of incentives to work hard because they were all forced to work collectively with egalitarian reward (see Lin, 1991), and the other is the Cultural Revolution starting in 1965 which compelled workers to shift their time and efforts from physical production to political movement. After a slight recovery in the 1970s, the economy started to grow rapidly at an exponential speed immediately after the initiation of market-orientated agricultural reform in 1978. China's experience confirms the well-known notion that institutions are indeed very important for a nation's economic performance.

The theoretical models in this paper, however, is only helpful for our understanding of the economy starting form 1978 and thereafter , so the institutions and human capital level in 1978 are simply taken as given initial conditions (captured by δ_0 and h_0, respectively). Although China's economic reform and institutional change were actually initiated spontaneously by a small group of poor peasants and local governments, only sometime later legalized and advocated by the central government, the central government is definitely the largest player with an overwhelming administrative power in terms of formulating and implementing new policies and formalizing institutional changes in the absence of free markets. In fact, ever since the Third Plenum of the Eleventh CCP National Congress held in 1978, the central and priority task of the central government has been shifted to the economic development, as can be clearly seen in all its subsequent five-year plans. This is to some extent consistent with our welfare maximization assumption in the theoretical central planner model.

Table 1 is also taken from Ravi Kanbur and Xiaobo Zhang, 2003, which highlights the secular trend of increasing openness and marketization along with the economic growth after 1978[11].In ourmodel, institutional barrier to learning externality comprises many aspects of the economy. It includes , for example, the institutional arrangement or policies that impede openness such as Tariff and policies toward FDI,etc. Of course, as is well documented in the literature, the institutional barriers can be very different for different countries. For instance,in some countries the largest barrier is the monopoly power of the incumbent domestic firms, which prevent more advanced foreign technology from being used [12] (see Parente and Prescott, 1994, 1999), while in some other countries, financial institutions are the most important barrier (Aghion, Howitt, and Mayer-Foulkes, 2005), etc..

China's economic reform is well-known for its piece-meal fashion as can be partly seen in the Table 1, much less radical than the reforms in Russia and some other East European and Latin American countries. China's overall economic performance after it began the institutional transition is widely appraised as successful, if not miraculous, on the other hand, some countries with radical reforms are doing quite well but some others suffered disasters. It is an extremely important yet very controversial welfare and policy issue how to set an optimal reform pace, and there has been heated debate both

[11]Please refer to their paper for a detailed explanation of the data and index construction method.

[12]At the current stage of our analysis, I don't differentiate the learning in human capital and technology spillover. In my model, only intangible capital like human capital (combined with inelastically supplied labor) will be needed to produce output, and hence technology and human capital are not fully differentiated.

Table 1

China: Economic Indicators, 1952-2000

Year	GDP Billion)	Import (Billion)	Total expenditure (Billion)	GOV (Billion)	Tariff rate (%)	Trade ratio (%)	Decentralization (%)	Industrialization (%)
1952	67.9	3.8	17.2	81.0	12.8	9.5	25.9	15.3
1953	82.4	4.6	21.9	96.0	11.0	9.8	26.1	17.5
1954	85.9	4.5	24.4	105.0	9.2	9.9	24.7	18.9
1955	91.0	6.1	26.3	110.9	7.6	12.1	23.5	19.7
1956	102.8	5.3	29.9	125.2	10.2	10.6	29.6	21.7
1957	106.8	5.0	29.6	124.1	9.6	9.8	29.0	25.5
1958	130.7	6.2	40.0	164.9	10.4	9.8	55.7	35.2
1959	143.9	7.1	54.3	198.0	9.9	10.4	54.1	43.8
1960	145.7	6.5	64.4	209.4	9.2	8.8	56.7	52.1
1961	122.0	4.3	35.6	162.1	14.5	7.4	55.0	37.7
1962	114.9	3.4	29.5	150.4	14.3	7.0	38.4	32.3
1963	123.3	3.6	33.2	163.5	11.6	6.9	42.1	33.5
1964	145.4	4.2	39.4	188.4	10.4	6.7	42.9	34.4
1965	171.6	5.5	46.0	223.5	10.3	6.9	38.2	30.4
1966	186.8	6.1	53.8	253.4	10.6	6.8	36.9	32.7
1967	177.4	5.3	44.0	230.6	7.3	6.3	38.7	28.1
1968	172.3	5.1	35.8	221.3	12.4	6.3	38.7	26.9
1969	193.8	4.7	52.6	261.3	13.5	5.5	39.3	31.7
1970	225.3	5.6	64.9	313.8	12.5	5.0	41.1	36.4
1971	242.6	5.2	73.2	348.2	9.5	5.0	40.5	39.5
1972	251.8	6.4	76.6	364.0	7.8	5.8	43.7	40.2
1973	272.1	10.4	80.9	396.7	8.7	8.1	44.4	39.9
1974	279.0	15.3	79.0	400.7	9.2	10.5	49.7	38.7
1975	299.7	14.7	82.1	446.7	10.2	9.7	50.1	40.2
1976	274.4	12.9	80.6	453.6	11.6	9.6	53.2	40.3
1977	320.2	13.3	84.4	497.8	19.8	8.5	53.3	41.9
1978	362.4	18.7	112.2	563.4	15.3	9.8	52.6	42.8
1979	403.8	24.3	128.2	637.9	10.7	11.3	48.9	41.3
1980	451.8	29.9	122.9	707.7	11.2	12.6	45.7	38.5
1981	486.0	36.8	113.8	758.1	14.7	15.1	45.0	34.5
1982	530.2	35.8	123.0	829.4	13.3	14.5	47.0	34.9
1983	595.7	42.2	141.0	921.1	12.8	14.4	46.1	36.1
1984	720.7	62.1	170.1	1083.1	16.6	16.7	47.5	37.0
1985	898.9	125.8	200.4	1333.5	16.3	23.0	60.3	38.6
1986	1020.1	149.8	220.5	1520.7	10.1	25.3	62.1	38.6
1987	1195.5	161.4	226.2	1848.9	8.8	25.8	62.6	38.7
1988	1492.2	205.5	249.1	2408.9	7.5	25.6	66.1	38.4
1989	1691.8	220.0	282.4	2855.2	8.3	24.6	68.5	39.4
1990	1859.8	257.4	308.4	3158.6	6.2	29.9	67.4	38.3
1991	2166.3	339.9	338.7	3478.2	5.5	33.4	67.8	41.5
1992	2665.2	444.3	374.2	4368.4	4.8	34.2	68.7	44.8
1993	3456.1	598.6	464.2	5939.8	4.3	32.6	71.7	49.7
1994	4667.0	996.0	579.3	8592.7	2.7	43.7	69.7	35.5
1995	5749.5	1104.8	682.4	11223.5	2.6	40.9	70.8	33.1
1996	6685.1	1155.7	793.8	12195.3	2.6	36.1	72.9	30.0
1997	7314.3	1180.7	923.4	13749.7	2.7	36.9	72.6	29.2
1998	7801.8	1162.2	1079.8	14320.5	2.7	34.4	71.1	27.0
1999	8206.8	1373.7	1318.8	15063.0	4.1	36.4	68.5	23.6
2000	8940.4	1863.9	1588.7	n.a	4.0	43.9	65.3	n.a.

in the academia and policy community. Our theoretical model claims that the welfare-maximizing institutional change could be gradual and carried out with multiple steps but it could also be radical and implemented in just one big step, all depending on the initial conditions, various parameters in the adjustment cost function and law of motions for the relative level of human capital, and whether there is sufficient capital and foreign aid, etc. Obviously, much more careful and thorough calibrations are needed in order to provide a more convincing and fruitful answer to these questions, but ourmodel clearly has the potential to reconcile all these apparently opposite possibilities simultaneously.

It's worth noticing that ourmodel is not necessarily restricted to the previously planned economies which are in the process of institutional transition to the market economy, nor does it have to be a poor economy, because our analysis has some crucial long-run results. For instance, our analysis with multiple adjustment in Section 3 shows that when facing the last adjustment option, the country might be never able to catch up with the most developed country if simply sticking on the passive learning externality of outside human capital because the barrier adjustment becomes increasingly costly while the benefit for the same adjustment size decreases as the economy grows, therefore at certain point, the human capital gap becomes so small that further barrier adjustment becomes unattractive as compared with the adjustment cost. This might help explain why Japan's economy was growing at a tremendous speed for several decades after World War II but the growth rates started to fall in the 1990s as its per capital GDP became close to or even exceeded US.

6 Some Theoretical Extensions

One natural variation is to consider the stochastic learning,for example, the learning speed can be assumed as:

$$dx(t) = I(x,t)dt + \sigma(x,t)dW,$$

where W is a standard Weiner Process. The previous analysis under certainty is simply a special case with $\sigma(x,t) = 0$. Hence the differences in the level and in the growth rates of the human capital (which is equivalent to the interpretation of GDP) between the two countries would fluctuate randomly rather than change monotonically or keep constant as in the deterministic model on and off Balanced Growth Path. We also need to specify the optimal Impulse Control Policy (ICP)specifying when to exercise a control and by how much.

When there is always sufficient financial resource to exercise a control, I hope that the problem is recursive enough so that the standard Hamilton-Jacoby-Bellman equations can be formulated. However, the difficulty lies in the fact that the although $x(t)$ follows , say, geometric Brownian motion, it is not a control variable and can't be adjusted immediately by its economic nature. Human capital level simply can't jump like a price in the price-setting monetary models , or jump as physical capital stock in the investment models. nor can the ratio of human capital jump. Hence the standard formulation is not directly applicable. I thus have to resort to the learning barrier variable, δ , which is the control variable capable of costly "jumping". So one possible form of the optimal ICP is to set a lower bound, b, such that δ needs to be adjusted downward whenever

$x(t)$ hits this lower bound, suggesting the learning is too slow presumably because the learning barrier becomes serious as $x(t)$ shrinks on average.(Typically, I assume $I(x,t)$ is decreasing,at least weakly, in $x(t)$). The convexity of the adjustment cost tends to make the adjustment more gradual, while the existence of fixed adjustment cost tends to make the adjustment intermittently. Particularly, it might imply that we need to find an optimal time-varying adjustment path(rather than a fixed optimal return point) for δ . We need to be cautious, however, to make sure the process is stationary and hence the problem has some "fixed point" in its optimal policy functions. For example, ourprevious assumption on the learning process in the deterministic environment adopted in section 3 doesn't work since it's not continuous. We might also need to find a better functional form for the adjustment cost function since there are too many discontinuities although it is perhaps one of the most convenient forms for deterministic setup. If the adjustment cost is covered by an outside agency, then the randomness of learning might make symmetric information and direct monitoring essential and perhaps indispensable to achieve the first best,as is quite different from the case under certainty, because the developing country ,as a beneficiary , might have strong incentive to report falsely about its usage of foreign aid (for example, by overconsumption), expecting to acquire more aid. The moral hazard problem becomes severe. If all the adjustment has to be financed by the developing country itself through saving, then the optimal adjustment path of the learning barrier has to be feasible at least on the expectation level. The problem obviously becomes more difficult.

A second important extension is to explicitly introduce the physical capital and time allocation by a representative consumer into the current model, while the central government simultaneously optimally adjusts the learning barrier subject to its budget constraint. As I emphasized in the paper, the budget constraint for the adjustment is an important issue. The central government finances its barrier adjustment by levying taxes or issuing bond, and then we might need to derive a dynamic tax scheme under different informational structures.

A third possible avenue is to take the positive approach and provide a more realistic political-economic micro-foundation for the formation of an adjustment plan. Either the assumption of the central government's benevolence is relaxed and replaced by a more "sophisticated " goal function , or we can model the process of producing a comprised barrier adjustment plan as a social choice problem for the democratic economies. A multi-agent setup is typically game-theoretic,perhaps dynamically. This will also make the adjustment cost function seem more concrete and endogenously determined. Dixit, Grossman and Gul (2000) seems very inspiring along this line.

7 Concluding Remarks

This paper builds a dynamic model trying to characterize how a Ramsey central government of a less developed economy should optimally adjust its learning barrier variable under complete or incomplete capital markets and how it affects its economic growth. Particularly, I provide a thorough analysis on what happens if only one adjustment is allowed in the two different scenarios of capital market completeness. A partial characterization is provided also for the multiple adjustment under complete capital market, which can be easily adapted to the analysis under incomplete market. Numerical experiment is the top priority on ouragenda for future research since I have shown that the explicit solutions for the value function and policy function are very hard to obtain even with only two opportunities to adjust, not to speak of the whole adjustment path with endogenously determined number of adjustment options. I also show in the appendix why our model can't be analyzed in the standard way in option theory since there exists no fixed point for the value function in the properly defined functional space. The extensions to uncertainty, dynamic taxation, and political process of barrier adjustment are briefly discussed.

Aside from the methodological part, I have also shown some nontrivial substantial messages delivered by the theoretical analysis, particularly, I show how this model might help us understand some important welfare and policy issues such as whether it is optimal for a transitional economy like China or Previous Soviet Union to undertake a radical reform or a gradual reform. Our analysis could also be useful to help us understand why some market economies like Japan experienced high growth in the early periods but became stagnant as it became richer.

Appendix: A Reformulation of the Model in Section 4.

In this appendix, I will show a reformulation of the model in section 4, trying to explain explicitly why it is hard to find a time-invariant Bellman equation and hence why no-fixed point of Bellman Equation exists in the properly defined functional space for the value function. Thus a direct analysis with a totally endogenous number of adjustment options is hard to obtain in this regular fashion. This is the reason why I tackle the problem step by step ,gradually increasing the exogenously given number of adjustment option in order to approximate the limiting case with unrestrained number of adjustment options.

The goal of the decision maker is still to maximize the total present discounted value of consumption same as in (5) subject to (2), (3), δ_0 and $t_0 = 0$ are given, and subject to the budget constraint, which can be rewritten (15) as follows:

$$S(t_{i+1}) + C(\delta_i, \delta_{i+1}) = S(t_i)e^{(t_{i+1}-t_i)r} + e^{t_{i+1}r}\int_{t_i}^{t_{i+1}} x(s)e^{-rs}ds, \forall i = 0, 1, 2, ... \tag{22}$$

and $S(t_i) \geq 0, \forall i = 0, 1, 2....$ Note (22) fully describes the evolution of saving for the whole time when we allow for infinite number of adjustments since for any non-stopping time t we can always define $t \equiv t_{i+1}$ for some i with $\delta_i = \delta_{i+1}$.Let τ be the next adjustment time immediately after the time point t, so there are four state variables in the value function:

$$V(\delta(t), x(t), S(t), t)$$
$$= \max_{\tau > t, \delta(\tau)} \left\{ S(t) + \int_t^\tau x(s)e^{-r(s-t)}ds - C(\delta(t), \delta(\tau))e^{-r(\tau-t)} + e^{-r(\tau-t)}V(\delta(\tau), x(\tau), S(\tau), \tau) \right\}, \forall t < \infty. \tag{23}$$

subject to

$$S(\tau) + C(\delta(t), \delta(\tau)) = S(t)e^{(\tau-t)r} + e^{\tau r}\int_t^\tau x(s)e^{-rs}ds, \forall t, \tag{24}$$

and

$$S(t) \geq 0, \forall t.$$

Note we need t as an indispensable fourth state variable because according to (2), $x(t)$ would be constant at certain period if the learning constraint remains binding for a while, so $x(t)$ together with $\delta(t)$ and $S(t)$ alone is not sufficient to denote all the possible different states. We might need t to track how long $x(t)$ stays constant. This is the whole reason why I don't have a time-invariant Bellman equation, intuitively because the total present discounted value will be always increasing as positive output is continuously produced although it has a finite upper bound.

Suppose at time $t^* < \infty$, the system arrives at the steady state at which $\delta(s) = \widehat{\delta}$, $x(s) = \frac{\eta}{\widehat{\delta}}, \forall s \geq t^*$, and $\tau = \infty$ since no more adjustment will be made. From (23), we have the present discounted value

$$e^{-t^*r}V(\widehat{\delta}, \frac{\eta}{\widehat{\delta}}, S(t^*), t^*) = e^{-t^*r}S(t^*) + \int_{t^*}^{\infty} \frac{\eta}{\widehat{\delta}} e^{-rs} ds, \tag{25}$$

where we use the previously established results that

$$V_0(\delta_0, x_0) \leq \lim_{\tau \to \infty} e^{-r\tau} V(\delta(\tau), x(\tau), S(\tau), \tau) \leq \frac{1}{r}.$$

There are numerous ways to allocate the consumption flow so long as its total discounted value equals (25) . One typical way is to set

$$c^*(s) = 0, \forall s < t^*, \text{ and } c^*(s) = \frac{\eta}{\widehat{\delta}} + rS(t^*), \forall s \geq t^*.$$

A trivial example is when $\delta_0 = \eta$ while $x_0 \leq 1$, hence no barrier adjustment will be ever made, in which case

$\widehat{\delta} = \delta_0 = \eta,\ t^* = T_0$ given by (7), and the total present discounted value (25) is given by (8).

We could define the value function in the discounted present value directly, but it still doesn't have a fixed point for exactly the same reason.

References

[1] Acemoglu, Daron, Simon Johnson, and James A. Robinson, 2001, "The Colonial Origins of Comparative Development: An Empirical Investigation", *American Economic Review* 91: 1369-1401

[2] Acemoglu, Daron and Simon Johnson, 2005, "Unbundling Institutitons", *Journal of Political Economy*,Vol.113(5):949-995

[3] Bruno, Michael, 1972, " Market Distortions and Gradual Reform", *Review of Economic Studies,*Vol.39(3): 373-383

[4] Buchanan, James M., 1975, " A Contractarian Paradigm for Applying Economic Theory," *American Economic Review* 74, No.2 Papers and Proceedings: 225-230

[5] ——-, 1987, "The Constitution of Economic Policy." *American Economic Review* 77,No. 3: 243-250

[6] Bulow, Jeremy, and Kenneth Rogoff.1989. "Sovereign Debt: Is to Forgive or Forget?" *American Economic Review* 79, No. 1:43-50

[7] Burnside, Craig and David Dollar, 2000, " Aid, Policies, and Growth", *American Economic Review* 90, No. 4: 847-868

[8] Dewatripont,M. and G. Roland, 1992, " The Virtues of Gradualism and Legitimacy in the Transition to the Market Economy", *Economic Journal*, Vol. 102 (411):291-300

[9] ——-,1992, "Economic Reform and Dynamic Political Constraints" , *Review of Economic Studies*, Vol. 59(4): 703-730

[10] Dixit, Avinash; Gene M Grossman, and Faruk Gul, 2000, "The Dynamics of Political Compromise", *Journal of Political Economy*,Vol 108 (3):531-568;

[11] Dixit, Avinash, 1998, *The Making of Economic Policy: A Transaction-Cost Political Perspective,*MIT Press

[12] Easterly, William, 2001, " The Lost Decades: Developing Countries' Stagnation in Spite of Policy Reform 1980-1998", *Journal of Economic Growth* 6(2):135-157

[13] Fischer, Stanley, 1991, "Economic Reform in USS and the Role of Aid", *Brookings Papers On Economic Activity*,Vol 1991(2):289-301

[14] Hall, Robert, and Charles Jones, 1999, " Why Do Some Countries Produce So Much More Output Per Worker Than Others? " *Quarterly Journal of Economics* 114:83-116

[15] Justin Yifu Lin, 1990, "Collectivization and China's Agricultural Crisis in 1959-1961", *Journal of Political Economy*, Vol.98 (6): 1228-52

[16] ———,2005, " How to Make Economic Policies Work in a Transitional Economy", *CCER working paper*

[17] Klenow, Peter J. , and Andres Rodriguez- Clare, 2004, " Externality and Growth", *NBER working paper* 11009;

[18] Lucas, Robert E. Jr., 1988. " On the Mechanics of Economic Development", *Journal of Monetary Economics*, 22:3-42;

[19] ——,1993, "Making a Miracle", *Econometrica*,61: 251-272;

[20] Murphy, Kevin M.; Andrei Shleifer; and Robert W . Vishny, 1992," The Transition to a Market Economy:Pitfalls of Partial Reform", *Quarterly Journal of Economics*, Vol.107, No.3 (August):889-906;

[21] Parente,Stephen L. and Edward Prescott, 1994, " Barriers to Technology Adoption and Development" , *Journal of Political Economy*,102: 298-321.

[22] ———, 2000, *Barriers to Riches*. Cambridge:MIT Press;

[23] Prescott, Edward C, 2002, " Prosperity and Depression", *American Economic Review*, 92:1-15.

[24] Persson, Torsten and Guido Tabellini, 2002, *Political Economics: Explaining Economic Policy,* MIT Press

[25] Romer, Paul, 1986, "Increasing Returns and Long-Run Growth." *Journal of Political Economy*, 94(October):1002-1037;

[26] Sachs, Jeffrey D., and Andrew Warner, 1995, "Economic Reform and the Process of Global Integration", *Brookings Papers on Economic Activity*: 1-118

[27] Stokey, Nancy L., 2005, Class Notes on Brownian Models in Economics, University of Chicago;

[28] Wu, Jinglian and Brue L. Reynolds, 1988, "Choosing a Strategy for China's Economic Reform",A*merican Economic Review, Papers and Proceedings*(May):461-466

[29] Kanbor, Ravi, and Xiaobo Zhang, 2003, *Fifty Year's Regional Inequality in China: A Journey Through Central Planning, Reform and Openness, WIDER working paper.*